WYTRZYMAŁOŚĆ KONSTRUKCJI I

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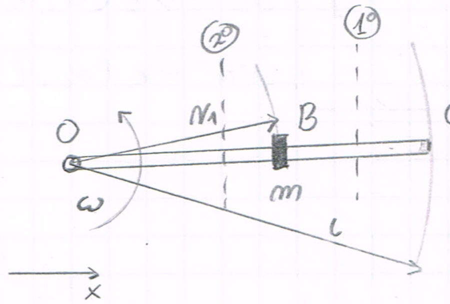
Zadanie 1.



$l = 100\,cm$ $m = 2\,kg$

$r_1 = 70\,cm$ $E = 2\cdot 10^5\,MPa$

$A = 10\,cm^2$ $\rho = 7{,}8\cdot 10^3\,kg/m^3$

$k_r = 100\,MPa$

od 1° $r \in (r_1, l\rangle$ $dN = dM\omega^2 x = \rho A dx \omega^2 x$

$$\int dN = \int_r^l \rho A \omega^2 x\,dx$$

$$N = \left[\rho A \omega^2 \frac{1}{2} x^2\right]_r^l$$

$$N_1(r) = \frac{\rho A \omega^2}{2}(l^2 - r^2)$$

$$\sigma_1(r) = \frac{1}{2}\rho\omega^2(l^2 - r^2)$$

od 2° $r \in \langle 0, r_1)$

$$\sigma_2(r) = \frac{1}{2}\rho\omega^2(l^2 - r^2) + \frac{m\omega^2 r_1}{A} \qquad N_2(r) = \frac{1}{2}\rho A\omega^2(l^2 - r^2) + m\omega^2 r_1$$

Naprężenia rosną w stronę punktu O.

$$\sigma_{2\,max} = k_r \quad , \quad \sigma_2(x) = \sigma_{2\,max} \Leftrightarrow r = 0$$

$$k_r = \omega_{max}^2\left(\frac{1}{2}\rho l^2 + \frac{m r_1}{A}\right)$$

$$\omega_{max} = \sqrt{\frac{k_r}{\frac{1}{2}\rho l^2 + \frac{m r_1}{A}}} = \sqrt{\frac{100\cdot 10^6\,Pa}{\frac{1}{2}\cdot 7{,}8\cdot 10^3\,\frac{kg}{m^3}\cdot(1m)^2 + \frac{2kg\cdot 0{,}7m}{10^{-3}m^2}}}$$

$$\omega_{max} \approx 137{,}36\,\frac{rad}{s}$$

$$n_{max}\,[rpm] = \omega_{max}\left[\frac{rad}{s}\right]$$

$$\boxed{n_{max} \approx 1312\,rpm}$$

$$N(r) = \begin{cases} 73{,}585(1-r^2) + 26{,}415 \\ 73{,}585(1-r^2) \end{cases} [kN] \quad \begin{matrix} \text{, dla } r \in \langle 0, r_1 \rangle \\ \text{, dla } r \in (r_1, L) \end{matrix}$$

$$\sigma(r) = \begin{cases} 73{,}585(1-r^2) + 26{,}415 \\ 73{,}585(1-r^2) \end{cases} [MPa] \quad \begin{matrix} \text{, dla } r \in \langle 0, r_1 \rangle \\ \text{, dla } r \in (r_1, L) \end{matrix}$$

$$\varepsilon(r) = \frac{\sigma(r)}{E}$$

$$u(r) = \int_0^r \varepsilon(r)\,dr = \frac{1}{E}\int_0^r \sigma(r)\,dr$$

dla $r \in \langle 0, r_1 \rangle$

$$u(r) = \frac{1}{E}\int_0^r \left(\frac{1}{2}\rho\omega_{max}^2(L^2 - r^2) + \frac{m\omega_{max}^2 r_1}{A}\right)dr$$

$$u(r) = \frac{1}{E}\left[\frac{1}{2}\rho\omega_{max}^2 L^2 r - \frac{1}{6}\rho\omega_{max}^2 r^3 + \frac{m\omega_{max}^2 r_1 r}{A}\right]_0^r$$

$$u(r) = \frac{\omega_{max}^2 r}{E}\left(\frac{1}{2}\rho\,(L^2 - \frac{1}{3}r^2) + \frac{m r_1}{A}\right)$$

dla $r \in (r_1, L\rangle$

$$u(r) = \int_0^{r_1}\varepsilon(r)\,dr + \int_{r_1}^r \varepsilon(r)\,dr$$

$$u(r) = \frac{\omega_{max}^2 r_1}{E}\left(\frac{1}{2}\rho(L^2 - \frac{1}{3}r_1^2) + \frac{m r_1}{A}\right) + \frac{1}{E}\int_{r_1}^r \left(\frac{1}{2}\rho\omega_{max}^2(L^2 - r^2)\right)dr =$$

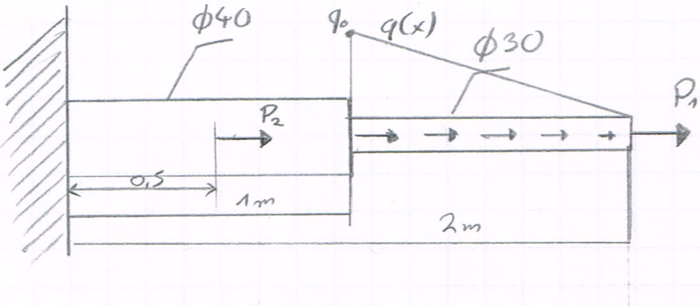
$$= \frac{\omega_{max}^2 r_1}{E}\left(\frac{1}{2}\rho(L^2 - \frac{1}{3}r_1^2) + \frac{m r_1}{A}\right) + \frac{\rho\omega_{max}^2}{2E}\left[L^2 r - \frac{1}{3}r^3\right]_{r_1}^r =$$

$$= \frac{\omega_{max}^2}{E}\left(\frac{1}{2}\rho(L^2 - \frac{1}{3}r^2)r + \frac{m r_1^2}{A}\right)$$

$$\boxed{u(r) = \begin{cases} \dfrac{\omega_{max}^2}{E}\left(\dfrac{1}{2}\rho r(L^2 - \dfrac{1}{3}r^2) + \dfrac{m r_1}{A}r\right) & \text{, dla } r \in \langle 0, r_1 \rangle \\ \dfrac{\omega_{max}^2}{E}\left(\dfrac{1}{2}\rho r(L^2 - \dfrac{1}{3}r^2) + \dfrac{m r_1^2}{A}\right) & \text{, dla } r \in (r_1, L\rangle \end{cases}}$$

$u_{całkowite} = u(L) \approx 0{,}3377$ mm

Zadanie 2.



$P_1 = P_2 = 100\,kN$

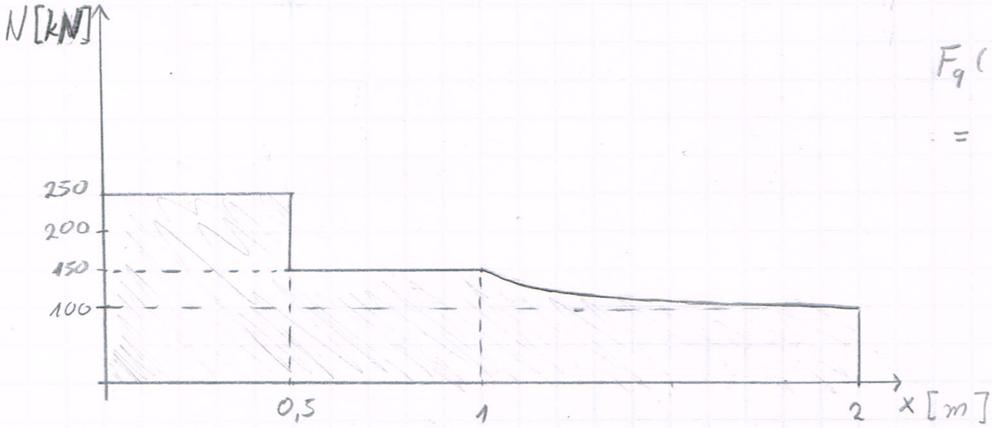
$q_0 = 100\,\frac{kN}{m}$

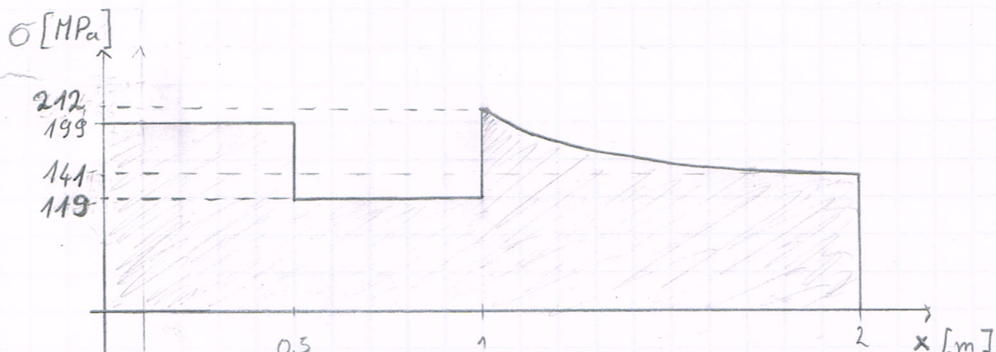
$E = 2\cdot 10^5\,MPa$

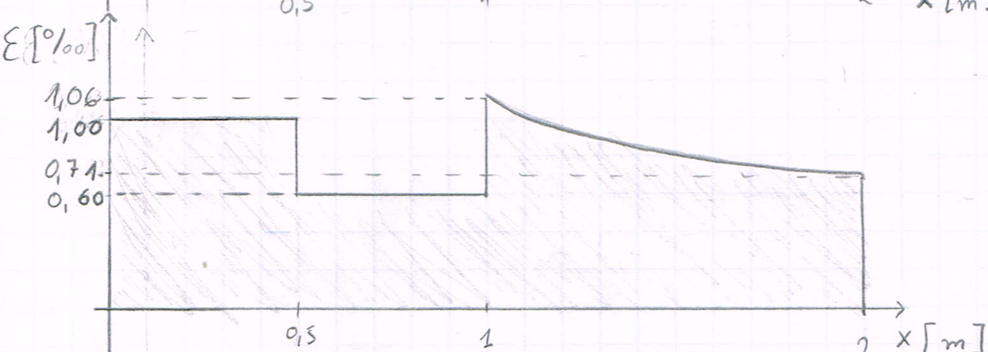
$q(x) = -q_0 x + 2q_0 \quad , x \in \langle 1,2 \rangle$

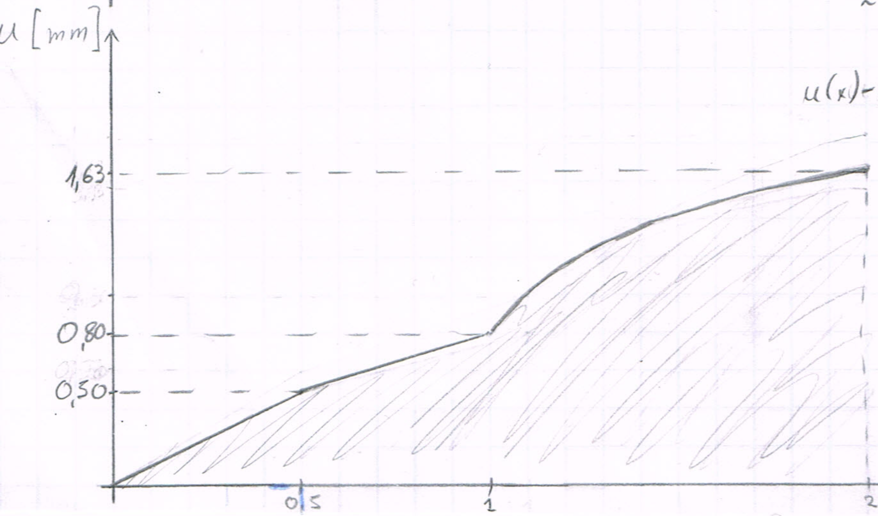
$F_q(x) = q_0\int_x^2 (2-x)\,dx = q_0\left[2x - \frac{1}{2}x^2\right]_x^2 =$

$= q_0\left(\frac{1}{2}x^2 - 2x + 2\right), \; x \in \langle 1, 2 \rangle$









$u(x) - u(1) = \int_1^x \frac{F_q(x) + P_1}{E\cdot A_2}\,dx = \frac{100}{EA_2}\int_1^x\left(\frac{1}{2}x^2 - 2x + 3\right)dx$

$= \frac{100kN}{EA_2}\left[\frac{1}{6}x^3 - x^2 + 3x\right]_1^x =$

$= \frac{100kN}{EA_2}\left(\frac{1}{6}x^3 - x^2 + 3x - \frac{13}{6}\right)\;[m]$

$, \text{ dla } \quad x \in \langle 1, 2 \rangle$